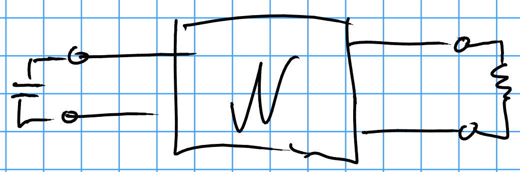
ST 2 - Tutorium, Blatt 5 - 8.06.2010

Lineare Schaltungen 2. Grades



$\underline{\dot{x}} = \underset{\sim}{A}\,\underline{x} + \underset{\sim}{B}\,v(t)$ => allgemeine Zustandsbeschreibung

1. Fall: $v \equiv 0$: homogene Form

$\underline{\dot{x}} = \underset{\sim}{A}\,\underline{x}$ $\qquad \underline{x}_{GGP} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

2. Fall: $v \equiv \text{const.}$: autonome Form

$\underline{\dot{x}} = \underset{\sim}{A}\,\underline{x} + \underset{\sim}{B}\,v(t) \iff \underset{\sim}{A}\,\underline{x} + \underset{\sim}{B}\,\underline{v}(t) = 0$

$\underset{\sim}{A}\,\underline{x} = -\underset{\sim}{B}\,v(t) \quad \big| \to \underset{\sim}{A}^{-1}$

$\boxed{\underline{x}_{GGP} = \underset{\sim}{A}^{-1}\underset{\sim}{B}\,v(t)} \quad (*)$

$\underline{x}' = \underline{x} - \underline{x}_{GGP}$

$\underline{\dot{x}}' = \underset{\sim}{A}(\underline{x} - \underline{x}_{GGP}) + \underset{\sim}{B}\,\underline{v}(t) = \underset{\sim}{A}\,\underline{x}'$

3. Fall $v = v(t)$: allgemeiner Erregung

Phasenportraits:

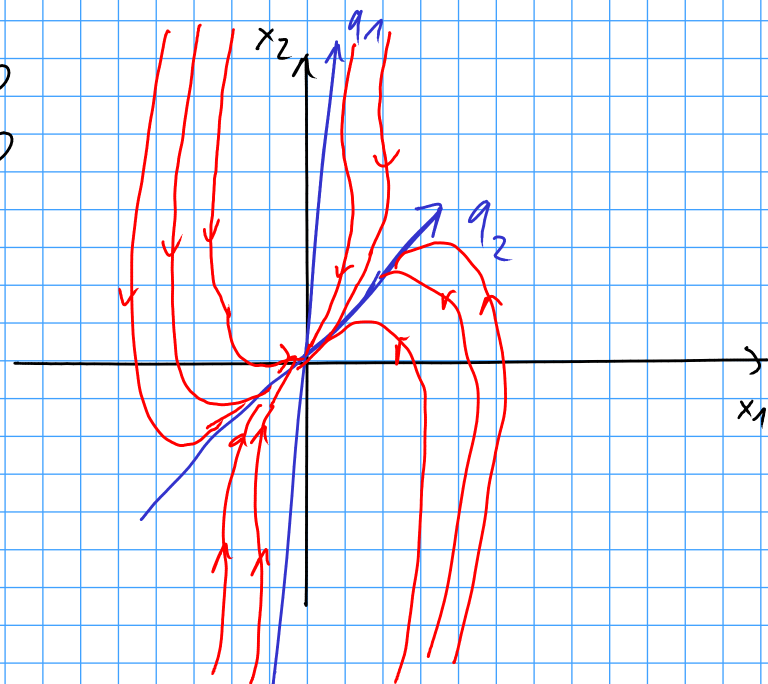
→ erste Unterscheidung: $\lambda_1, \lambda_2 \in \mathbb{R}$ oder $\lambda_1, \lambda_2 \in \mathbb{C}$

1. Fall: $\lambda_1, \lambda_2 \in \mathbb{R}$

→ stabilen Knoten: $\lambda_1 < 0$
$\lambda_2 < 0$

o.B.d.A. $|\lambda_1| > |\lambda_2|$

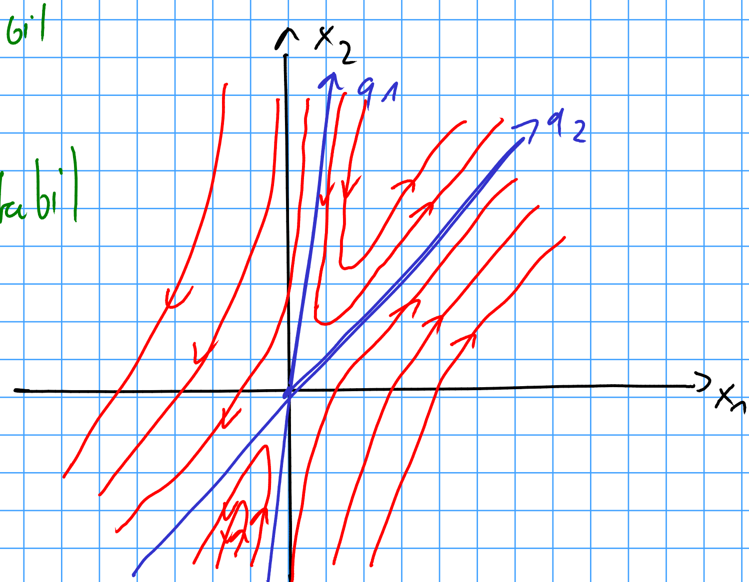
$\downarrow$ $\downarrow$
q_1 q_2



→ instabilen Knoten: $\lambda_1 > 0$
$\lambda_2 > 0$

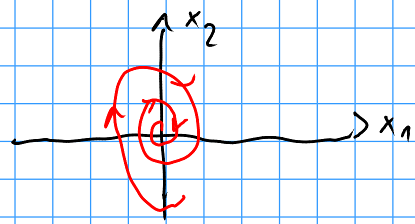
→ Sattelpunkt: $\lambda_1 < 0$ → stabil
$\lambda_2 > 0$ ← instabil

q_1 ist Eigenvektor zum stabilen Eigenwert, gibt also Herkunft der Trajektorie an

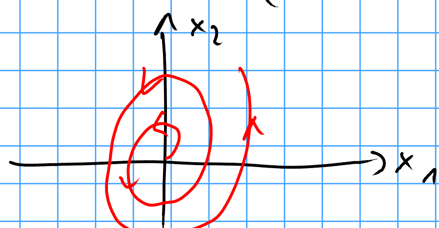


Fall 2: imaginäre Eigenwerte

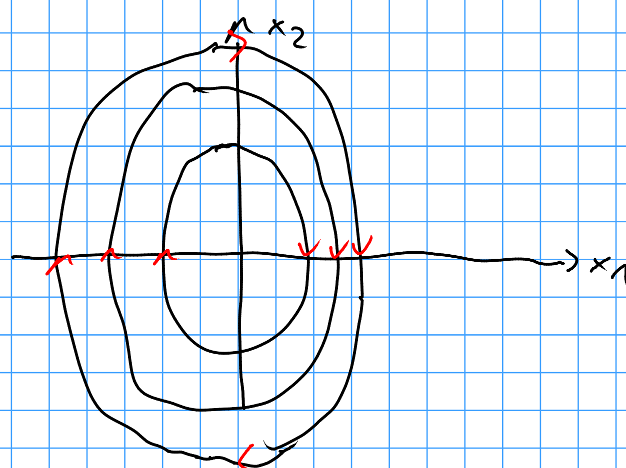
→ stabile Strudel: $Re(\lambda_1) < 0$, $Re(\lambda_2) < 0$



→ instabiler Strudel: $Re(\lambda_1) > 0$, $Re(\lambda_2) > 0$



→ Wirbel: $Re(\lambda_1) = Re(\lambda_2) = 0$



Aufgabe 1

1. Sattelpunkt: $\underline{x}_{GGP} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$q_1 = \begin{pmatrix} -2 \\ -1 \end{pmatrix} \hat{=} \begin{pmatrix} 2 \\ 1 \end{pmatrix} \qquad q_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

2. $x = \underset{\sim}{e}^{tA} \underline{x}_0$

Matrixexponential-funktion

$$A = Q \Lambda Q^{-1} \quad ; \quad e^{tA} = \sum_{k=0}^{\infty} \frac{(tA)^k}{k!} = \sum_{k=0}^{\infty} \frac{[t(Q\Lambda Q^{-1})]^k}{k!}$$

$$= Q e^{t\Lambda} Q^{-1}$$

$$\underline{x} = Q e^{t\Lambda} Q^{-1} \underline{x}_0 \quad \Big| \to Q^{-1}$$

$$\underbrace{Q^{-1} \underline{x}}_{} = e^{t\Lambda} \underbrace{Q^{-1} x_0}_{}$$

$$\underline{\xi} = e^{t\Lambda} \underline{\xi}_0$$

$$\xi_0 = \underset{\sim}{Q}^{-1} \underline{x}_0 \quad \text{mit} \quad Q = (q_1 \;\; q_2) = \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$$

$$Q^{-1} = \frac{1}{3} \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}$$

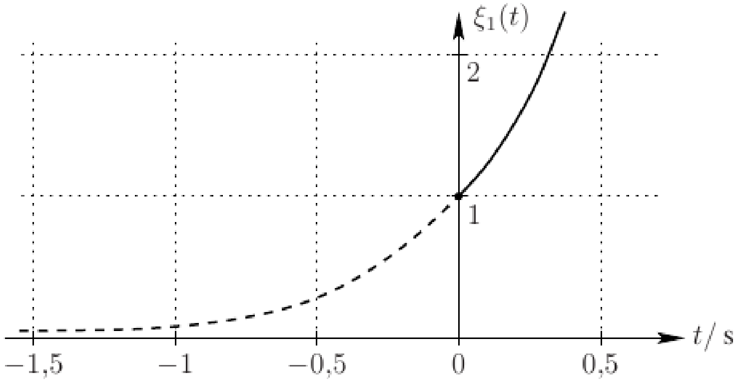
$$\xi_0 = \frac{1}{3} \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 0 \\ -3 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 0 \\ -1 \end{pmatrix}}}$$

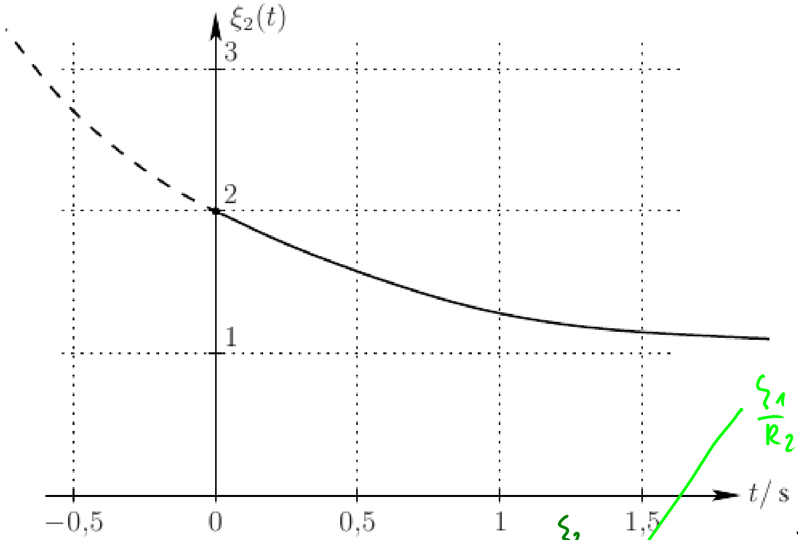
3. (siehe Beiblatt)

4. $\underline{q}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\underline{q}_2 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$\lambda_1 = -4 s^{-1}$ $\lambda_2 = -0{,}5 s^{-1}$

$\Rightarrow$ siehe Beiblatt





$\frac{\xi_1}{R_2} + \frac{\xi_2}{R_1}$

$-\left(\frac{\xi_2}{R_1} + \frac{\xi_1}{R_2}\right) \cdot 1k\Omega = -x_1$

$= -(\xi_1(t) + 2\xi_2(t))$

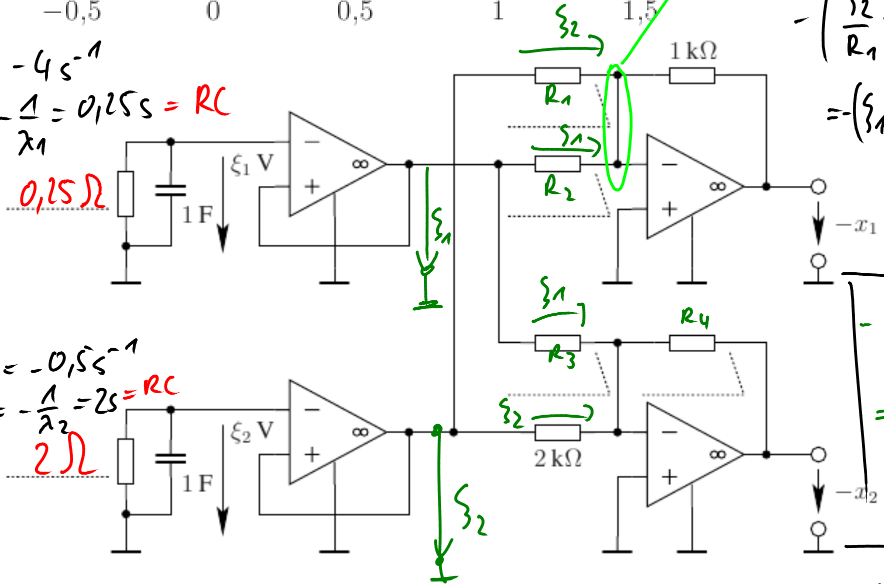
$R_2 = 1k\Omega$

$R_1 = 0{,}5k\Omega$

$\lambda_1 = -4\,s^{-1}$

$\tau = -\frac{1}{\lambda_1} = 0{,}25\,s = RC$

$0{,}25\,\Omega$



$-\left(\frac{\xi_1}{R_3} + \frac{\xi_2}{2k\Omega}\right) \cdot R_4 = -x_2 =$

$= -(2\xi_1(t) + 3\xi_2(t))$

$R_4 = 6k\Omega$

$R_3 = 3k\Omega$

$\lambda_2 = -0{,}5\,s^{-1}$

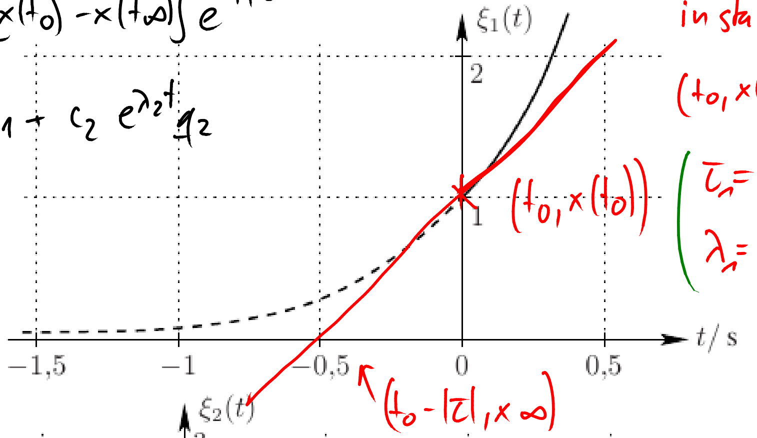
$\tau_2 = -\frac{1}{\lambda_2} = 2s = RC$

$2\,\Omega$

Rücktrafo: $x = Q\xi(t) = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} \xi_1(t) \\ \xi_2(t) \end{pmatrix} = \begin{pmatrix} \xi_1(t) + 2\xi_2(t) \\ 2\xi_1(t) + 3\xi_2(t) \end{pmatrix}$

$$x(t) = x(t_\infty) + [x(t_0) - x(t_\infty)]\, e^{-t/\tau}$$

$$\underline{x}(t) = c_1 e^{\lambda_1 t} \underline{g}_1 + c_2 e^{\lambda_2 t} \underline{g}_2$$

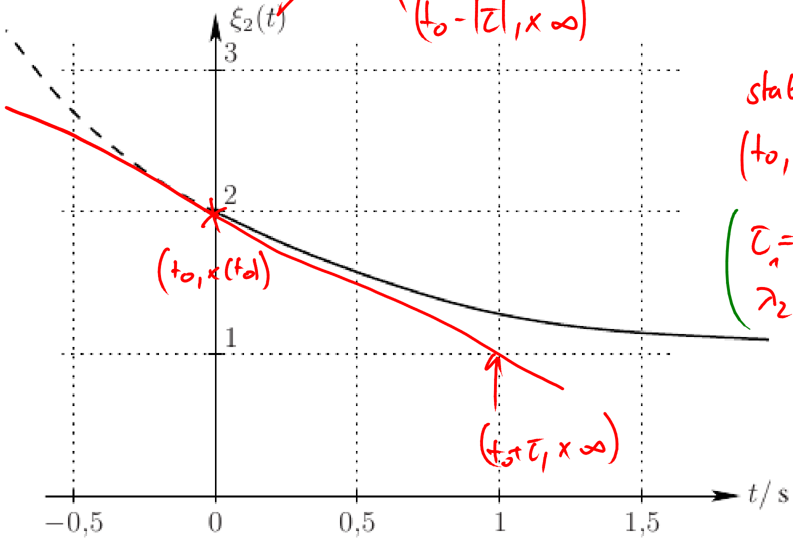


instabiler Fall:

$(t_0, x(t_0)),\ (t_0 - |\tau|, x_\infty)$

$\tau_1 = -0{,}5$

$\lambda_1 = -\frac{1}{\tau} = \underline{\underline{2}} > 0$



stabiler Fall:

$(t_0, x(t_0)),\ (t_0 + \tau, x_\infty)$

$\tau_1 = 1$

$\lambda_2 = -\frac{1}{\tau_1} = -1$