EMF-Tutorübung, Blatt 1, 2.11.2010

WICHTIGES: http://fabis-site.net/uni/emf/ [em] Mitschriften, Zusammenfassungen, [FS (???)]

NÄCHSTER TERMIN: Fr, 16.30-18.00

Wiederholung: Maxwell'sche Gleichungen in differentieller u. integraler Form

- diff. Form:

$$\operatorname{div} \vec{D} = \rho$$ ... Gaußsches Gesetz...

$$\operatorname{div} \vec{B} = 0$$ ... Quellenfreiheit B-Feld...

$$\operatorname{rot} \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$ ... Faradaysches Induktionsgesetz...

$$\operatorname{rot} \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$$ ... Ampèresches Durchflutungsgesetz...

- int. Form:

$$\iint_{\partial V} \vec{D}\, d\vec{a} = \iiint_V \rho \, dV = Q(V)$$

$$\iint_{\partial V} \vec{B}\, d\vec{a} = 0$$

$$\int_{\partial A} \vec{E}\, d\vec{r} = -\frac{\partial}{\partial t} \iint_A \vec{B}\, d\vec{a}$$

$$\int_{\partial A} \vec{H}\, d\vec{r} = \iint_A j\, d\vec{a} + \iint_A \frac{\partial \vec{D}}{\partial t}\, d\vec{a}$$

Materialgleichungen: (isotrope, homogene Medien)

$$\left.\begin{aligned} \vec{D} &= \varepsilon \vec{E} \\ \vec{B} &= \mu \vec{H} \\ \vec{j} &= \sigma \vec{E} \end{aligned}\right\}$$ hier: ε, μ, σ skalare Größen, keine Tensoren

Bücher:

- Menke: Elektromagnetische Felder
- Electromagnetic's Explained, R. Schmitt
- Student's Guide to Maxwell's Equations

⇒ Elektrische Energiedichte

$$W_{el} = \frac{1}{2} \frac{1}{4\pi\varepsilon} \sum_{\substack{i \neq k \\ i,k=1}}^{N} \frac{q_i q_k}{|\vec{r}_k - \vec{r}_i|} = [q\,\phi]$$

$$\downarrow$$

$$w_{el} = \int_0^{\vec{D}} \vec{E}(\vec{D}')\, d\vec{D}' \quad \Longleftrightarrow \quad \delta w_{el} = \vec{E} \cdot \delta\vec{D}$$

Spezialfall: ε skalar, Material homogen: $\vec{D} = \varepsilon \vec{E}$

$$w_{el} = \int_0^{\vec{D}} \frac{\vec{D}'}{\varepsilon} \, d\vec{D}' = \frac{1}{\varepsilon} \left[\frac{1}{2} |\vec{D}'|^2 \right]_0^{\vec{D}} = \frac{1}{2\varepsilon} |\vec{D}|^2 =$$

$$= \frac{1}{2} \vec{E} \cdot \vec{D}$$

Magnetische Energiedichte:

$$P_{elmag} = -\sum_{k=1}^{N} \vec{F}_k(\vec{r}_k) \cdot \vec{v}_k = -\sum_{k=1}^{N} q_k \Big(\vec{E}_k + \underbrace{\vec{v}_k \times \vec{B}_k) \cdot \vec{v}_k}_{=0}$$

$$= -\sum_{k=1}^{N} q_k \vec{E}_k \vec{v}_k$$

$$w_{mag} = \int_0^{\vec{B}} \vec{H}(\vec{B}') \, d\vec{B}' \iff \delta w_{mag} = \vec{H} \cdot \delta \vec{B}$$

Spezialfall: μ Skalar $\quad \vec{B} = \mu \vec{H}$

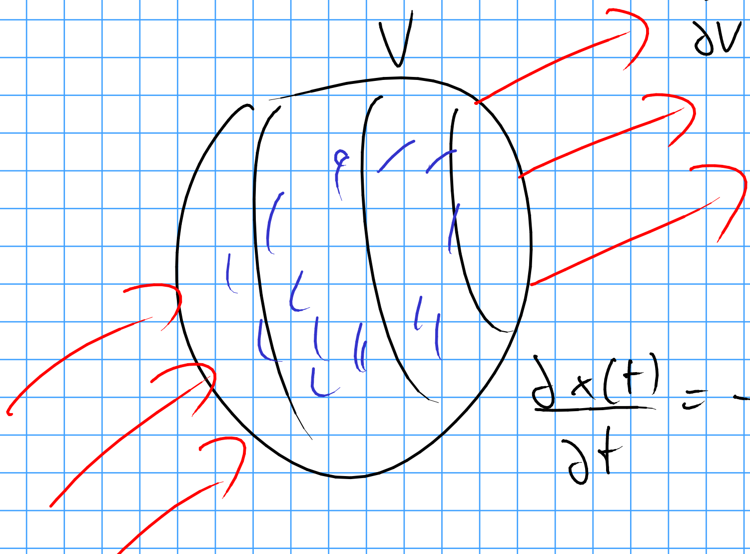
$$w_{mag} = \int_0^{\vec{B}} \frac{\vec{B}'}{\mu} \, d\vec{B}' = \frac{1}{2\mu} |\vec{B}|^2 = \frac{1}{2} \vec{B} \cdot \vec{H}$$

$\Rightarrow$ Poynting-Vektor: $\vec{S} = \vec{E} \times \vec{H}$ $\Rightarrow$ elektromagnetische Energiestromdichte

Bilanzgleichungen:

X physikalische größe; $X(V) = \iiint_V x(\vec{r}, t)\, dV$

$$\frac{dX(V)}{dt} = -\int_{\partial V} \vec{j}_x \, d\vec{a} + \int_V \Pi_x \, dV$$



Produktionsrate von X

$$\iint_{\partial V} \vec{u} \, d\vec{a} = \iiint_V \operatorname{div} \vec{u} \, dV$$

Satz v. Gauß

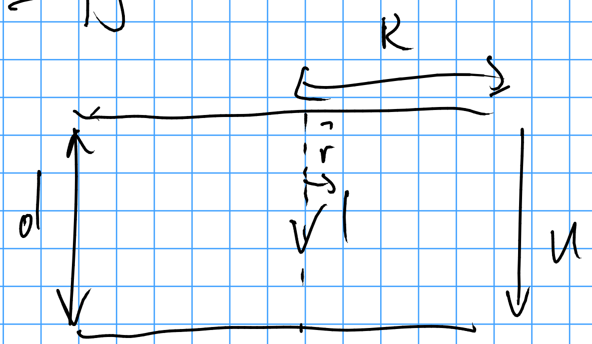
$$\frac{\partial x(t)}{\partial t} = -\operatorname{div} \vec{j}_x + \Pi_x$$

Bilanzgleichung d. Ladungserhaltung:

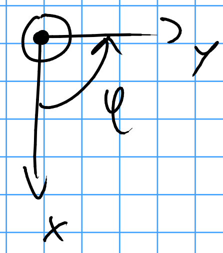
$$\operatorname{div} \vec{j} + \frac{\partial \rho}{\partial t} = 0$$

$$\iint_{\partial V} \vec{S} \, d\vec{a} = -P_{verlust} = -\iiint_V p_{verlust} \, dV$$

Aufgabe 1



geg: R, d



a) ges: $\vec{S}$

Lös: $\vec{S} = \underline{\underline{\vec{E}}} \times \underline{\underline{\vec{H}}}$ mit $\vec{E}$: $U = \int_C \vec{E}\, d\vec{r} = \int_0^d -E_z \cdot \vec{e}_z \cdot \vec{e}_z \, dz$

$$= -E_z \cdot d$$

$$\Rightarrow \vec{E} = -\frac{U}{d} \cdot \vec{e}_z$$

$$\int_{\partial A} \vec{H}\, d\vec{r} = \iint_A \vec{j}\, d\vec{a} = I$$

$$\int_0^{2\pi} H_\varphi \cdot (-\vec{e}_\varphi) \cdot \vec{e}_\varphi \, r\, d\varphi = I \quad \Rightarrow \quad H_\varphi = -\frac{I}{2\pi r}$$

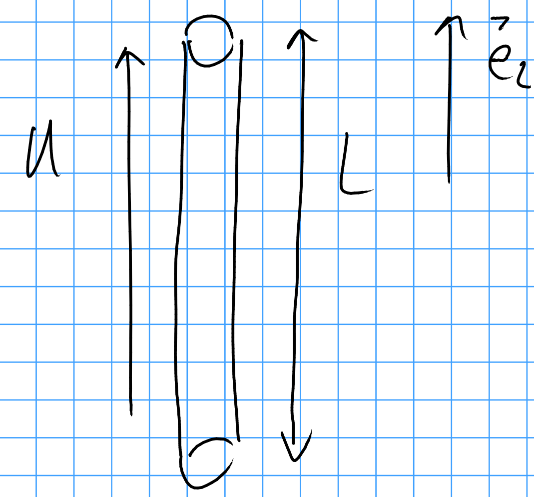
$$\vec{S} = -\frac{U}{d} \cdot \vec{e}_z \times -\frac{I}{2\pi r} \vec{e}_\varphi = -\frac{U \cdot I}{2\pi r d} \vec{e}_r$$

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

0 0 0 | 0 1

b) $\iint_{\partial V} \vec{S}\, d\vec{a} = \int_0^d \int_0^{2\pi} -\frac{U \cdot I}{2\pi r d} \vec{e}_r \, \underline{\vec{e}_r\, r\, d\varphi\, dz} = -U \cdot I = -P$

Aufgabe 3



geg: σ

a) ges: $\vec{S}$

Lös: $\vec{S} = \vec{E} \times \vec{H}$

mit $\vec{E} = \frac{U}{L} \vec{e}_z$

$$I(r) = \iint_A \vec{j}\, d\vec{a} = \iint_A \sigma \cdot \vec{E}\, d\vec{a} = \int_0^{2\pi}\int_0^{r} \sigma \cdot \frac{U}{L} \vec{e}_z \cdot \vec{e}_z\, r'\, dr'\, d\varphi$$

$$= \sigma \frac{U}{L} \cdot 2\pi \cdot \frac{1}{2} r^2 = \sigma \frac{U}{L} \pi r^2$$

$$\vec{H} = \frac{I(r)}{2\pi r} \vec{e}_\varphi = \frac{\sigma \frac{U}{L} \cancel{\pi} r^{\cancel{2}}}{2 \cancel{\pi} \cancel{r}} \vec{e}_\varphi \quad \sigma \frac{U}{L} \frac{1}{2} r \cdot \vec{e}_\varphi$$

$$R = \rho \cdot \frac{l}{A} = \frac{1}{\sigma} \cdot \frac{l}{A} \qquad I = \frac{U}{R(r)} = \frac{1}{\sigma} \frac{L}{r^2 \pi}$$

$$\vec{S} = \vec{E} \times \vec{H} = \frac{U}{L} \vec{e}_z \times \sigma \frac{U}{L} \frac{1}{2} r \cdot \vec{e}_\varphi = -\frac{1}{2} \sigma \frac{U^2}{L^2} r\, \vec{e}_r$$

b) $\operatorname{div} \vec{S} + \underbrace{\frac{\partial w_{elmag}}{\partial t}}_{} \overset{!}{=} -p_{verlust}$

↙ Satz v. Gauß

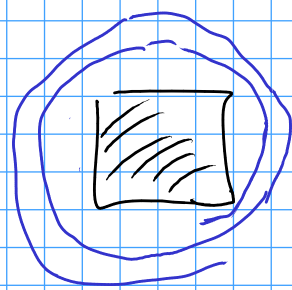
$$\oint_{\partial V} \vec{S}\, d\vec{a} = \int_0^{L}\int_0^{2\pi = 0} -\frac{1}{2} \sigma \frac{U^2}{L^2} r\, \vec{e}_r \cdot \vec{e}_r \cdot r\, d\varphi\, dz = -\frac{1}{2} \sigma \frac{U^2}{L} r^2 \cdot 2\pi$$

$$= \sigma \frac{U^2}{L} r^2 \pi$$

$$U \cdot I = U \cdot \sigma \frac{U}{L} \pi r^2 = \frac{U^2}{L} \sigma \pi r^2$$

Aufgabe 2

Querschnitt:



N Umwicklungen

geg: $I, N, \mu_0 \mu_r$

a) ges: $H(r), B(r)$

Lös: Ansatz wie vorher, jedoch Berücksichtigung der N Wicklungen

$$\int_{\partial A} \vec{H} \, d\vec{r} = N \cdot I$$

$$\Rightarrow \quad H_\varphi(r) = \frac{N \cdot I}{2\pi r} \qquad \vec{B} = \mu \vec{H}$$

$$\Rightarrow \quad B_\varphi(r) = \mu_0 \mu_r \frac{NI}{2\pi r}$$

b) ges: W_{mag}

Lös: $w_{mag} = \frac{1}{2} \vec{H} \cdot \vec{B} = \frac{1}{8} \left(\frac{NI}{\pi r}\right)^2 \mu_0 \mu_r$

c) $$W_{mag} = \iiint_V w_{mag} \, dV = \int_0^a \int_0^{2\pi} \int_{d/2}^{d/2+b} \frac{1}{8} \left(\frac{NI}{\pi r}\right)^2 \mu_0 \mu_r \, r \, dr \, d\varphi \, dz$$

$$= \frac{1}{8} \frac{(NI)^2}{\pi^2} \cdot 2\pi a \mu_0 \mu_r \left[\ln r\right]_{d/2}^{d/2+b} = \frac{1}{4} \frac{NIa}{\pi} \ln\left(1 + \frac{2b}{d}\right) \mu_0 \mu_r$$